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## ACOUSTIC INTENSITY VECTORS FROM AN INFINITE PLATE WITH LINE ATTACHMENTS

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# ACOUSTIC INTENSITY VECTORS FROM AN INFINITE PLATE WITH LINE ATTACHMENTS

BY

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Summary

Formulae are given for the acoustic pressure and particle velocities from which the acoustic intensity vectors are calculated. Plots of the vectors illustrate their utility as a visual aid to the understanding of the physics of fluid-structure interaction.



18 pages
7 figures

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## LIST OF SYMBOLS

| | |
|---|---|
| $(x,z)$ | Cartesian coordinates |
| $W(x)$ | plate displacement |
| $W(x,z)$, $U(x,z)$ | fluid particle displacements |
| $\dot{W}(x,z)$, $\dot{U}(x,z)$ | fluid particle velocities |
| $p(x,z)$, $p^*(x,z)$ | acoustic pressure and its complex conjugate |
| $\overline{W}(\alpha)$, $\overline{p}(\alpha,z)$ | Fourier transforms of plate displacement and pressure |
| $D$ | flexural rigidity of plate, $=Eh^3/12(1-\nu^2)$ |
| $E$, $\eta$ | Young's modulus and loss factor |
| $\nu$ | Poisson's ratio |
| $\rho_s$, $h$ | density and thickness of plate |
| $\rho$, $c$ | density and sound velocity of fluid |
| $f$ | frequency of line force, Hz |
| $\omega$ | radian frequency, $=2\pi f$ |
| $k$ | acoustic wavenumber, $=2\pi f/c$ |
| $F_o$ | magnitude of line force |
| $\delta$ | Dirac delta function |
| $I_x(x,z,t)$, $I_z(x,z,t)$ | instantaneous acoustic intensities |
| $\overline{I}_x(x,z)$, $\overline{I}_z(x,z)$ | time averaged acoustic intensities |
| $M$, $S$ | mass and spring constant |
| $\{W\}$ | $n_x1$ displacement matrix |
| $\{F\}$, $\{P\}$ <br> $n_x1$ $n_x1$ | $n_x1$ force matrices |
| $[D]$, $[D]^{-1}$ <br> $n_xn$ $n_xn$ | $n_xn$ receptance matrix and its inverse |
| $[A]$ <br> $n_xn$ | $n_xn$ diagonal matrix with elements $a_{ii}$ |

## INTRODUCTION

The time averaged acoustic intensity vector diagrams not only provide concise visual information about the magnitude and direction of the energy flow in an acoustic fluid, but also serve as an aid to the understanding of the complex physics involved in fluid-structure interactions. Acoustic intensity vectors can be determined either experimentally or theoretically. Fahy [1] has used a prototype intensity meter to measure the intensity distribution around a small machine, thus demonstrating the practicality of measurement. Other authors [2,3] have suggested procedures for measuring the acoustic intensity close to a vibrating surface. Williams et al [4] have constructed a microphone array of 256 elements, which is used in the nearfield of a vibrator: subsequent computer processing enables reconstruction of the acoustic field and particle velocities, and hence acoustic intensity vectors. Kristiansen [5] has produced theoretical vectors due to assumed mode shapes of a membrane vibrating in an infinite baffle.

Here, acoustic intensity vectors from an elastic plate with line attachments are derived theoretically. The line attachments may be time-harmonic forces, masses, masses and springs, grounded springs or displacement constraints. In Section 2 formulae are given for the acoustic pressure and particle velocities due to line force excitation of the plate. The acoustic intensity vector is defined in Section 3. The procedure necessary to calculate the dynamic stiffness of the plate with line attachments is given in Section 4, where the equivalent forces due to the line attachments are also calculated. Finally, in Section 5, acoustic intensity vector plots for forces and constraints are discussed.

## 2. ACOUSTIC PRESSURE AND DISPLACEMENT DUE TO LINE FORCE

A uniform infinite thin plate occupies the plane $z=0$, and it is subjected to a prescribed time-harmonic line force, $F_0\exp(-i\omega t)$, located at $x=x_0$. One side of the plate is in contact with an acoustic fluid filling the half-space $z>0$, and the other side contacts a vacuum. The geometry of interest is shown in Figure 1. Time variation, $\exp(-i\omega t)$, will be omitted from all equations.

The plate displacement and fluid sound pressure are expressed as Fourier transforms:

$$W(x) = (1/2\pi)\int_{-\infty}^{\infty} \overline{W}(\alpha)\exp(i\alpha x)d\alpha \tag{1}$$

$$p(x,z) = (1/2\pi)\int_{-\infty}^{\infty} \overline{p}(\alpha,z)\exp(i\alpha x)d\alpha \tag{2}$$

Substitute these transforms into the plate and acoustic equations of motion

$$[D\partial^4/\partial x^4-\omega^2\rho_s h]W(x) = F_0\delta(x-x_0)-p(x,0) \tag{3}$$

$$(\partial^2/\partial x^2+\partial^2/\partial z^2)p(x,z) = -k^2p(x,z) \tag{4}$$

to give, after carrying out some straightforward algebra,

$$W(x) = (F_0/2\pi)\int_{-\infty}^{\infty} \frac{\exp[i\alpha(x-x_0)]\,d\alpha}{D\alpha^4-\omega^2\rho_s h-i\rho\omega^2/\gamma} \tag{5}$$

$$p(x,z) = (-i\rho\omega^2 F_0/2\pi)\int_{-\infty}^{\infty} \frac{\exp(i\gamma z)\exp[i\alpha(x-x_0)]\,d\alpha}{\gamma(D\alpha^4-\omega^2\rho_s h-i\rho\omega^2/\gamma)} \tag{6}$$

where $\gamma = \sqrt{(k^2-\alpha^2)}$ with $\mathrm{Im}(\gamma)>0$ in order to satisfy the radiation condition. It is to be noted that the normal fluid particle displacement (cf Eq. 7) at the plate surface is equal to the plate displacement. This is the boundary condition of normal displacement continuity.

The acoustic particle displacements in the fluid are given by the equations

$$\rho\omega^2[U(x,z),W(x,z)] = [\partial p(x,z)/\partial x, \partial p(x,z)/\partial z] \tag{7}$$

which become, on making use of Eq. 6,

$$U(x,z) = (F_0/2\pi)\int_{-\infty}^{\infty} \frac{\alpha.\exp(i\gamma z)\exp[i\alpha(x-x_0)]\,d\alpha}{\gamma(D\alpha^4-\omega^2\rho_s h-i\rho\omega^2/\gamma)}$$

$$W(x,z) = (F_0/2\pi)\int_{-\infty}^{\infty} \frac{\exp(i\gamma z)\exp[i\alpha(x-x_0)]\,d\alpha}{D\alpha^4-\omega^2\rho_s h-i\rho\omega^2/\gamma} \tag{8}$$

The acoustic pressure and fluid particle displacements due to a number of line forces can be found by linear superposition.

## 3. ACOUSTIC INTENSITY VECTORS

Let $p(x,z)$, $U(x,z)$ and $W(x,z)$ be respectively the complex pressure, fluid particle x-displacement and fluid particle z-displacement due to externally applied line forces. The fluid particle velocities are derived by time-differentiating the displacements. They are

$$\dot{U}(x,z) = -i\omega U(x,z)$$

$$\dot{W}(x,z) = -i\omega W(x,z) \tag{9}$$

The instantaneous acoustic intensities are defined as

$$I_x(x,z,t) = \mathrm{Re}[p(x,z)\exp(-i\omega t)].\mathrm{Re}[\dot{U}(x,z)\exp(-i\omega t)]$$

$$I_z(x,z,t) = \mathrm{Re}[p(x,z)\exp(-i\omega t)].\mathrm{Re}[\dot{W}(x,z)\exp(-i\omega t)] \tag{10}$$

The time averaged acoustic intensities are given by

$$\bar{I}_x(x,z) = \lim_{T\to\infty} \frac{1}{T}\int_0^T I_x(x,z,t)dt$$

$$\bar{I}_z(x,z) = \lim_{T\to\infty} \frac{1}{T}\int_0^T I_z(x,z,t)dt \tag{11}$$

which reduce to the formulae

$$\bar{I}_x(x,z) = \tfrac{1}{2}\mathrm{Re}[p^*(x,z)\dot{U}(x,z)]$$

$$\bar{I}_z(x,z) = \tfrac{1}{2}\mathrm{Re}[p^*(x,z)\dot{W}(x,z)] \tag{12}$$

The amplitude and direction of the resultant acoustic intensity vector are respectively

$$\bar{I}(x,z) = \sqrt{[\bar{I}_x{}^2(x,z) + \bar{I}_z{}^2(x,z)]}$$

$$\theta = \tan^{-1}(\bar{I}_z(x,z)/\bar{I}_x(x,z)) \tag{13}$$

## 4. EQUIVALENT FORCES DUE TO LINE ATTACHMENTS

### (a) General

The plate dynamic stiffness equation must first be formed. From this, the system dynamic stiffness equation is then derived using standard finite element methods. This system equation is solved to find the plate displacements, which then yield the equivalent forces.

### (b) Plate Dynamic Stiffness Equation

Let $n$ separate line forces, $\{F\}$, act on the plate at the points $x_i$, $i=1,n$. Then the $n$ plate displacements, $\{W\}$, are related to these forces by the frequency dependent receptance matrix, $[D]$, where

$$\underset{n\times 1}{\{W\}} = \underset{n\times n}{[D]} \cdot \underset{n\times 1}{\{F\}} \tag{14}$$

The $ij^{th}$ element of $[D]$ is simply the response at $x_i$ due to unit force at $x_j$. It is given by Eq. 5. The inverse of Eq. 14 is the plate dynamic stiffness equation.

$$\underset{n\times n}{[D]^{-1}} \cdot \underset{n\times 1}{\{W\}} = \underset{n\times 1}{\{F\}} \tag{15}$$

(c) System Dynamic Stiffness Equation

The system dynamic stiffness equation is of the form

$$\underset{n\times n}{[D^{-1} + A]} \; . \; \underset{n\times 1}{\{W\}} = \underset{n\times 1}{\{P\}} \tag{16}$$

where $[A]$ is a diagonal matrix whose elements are derived from the mass/spring attachments, and $\{P\}$ is a column matrix of prescribed line forces. The mass/spring attachments are incorporated as follows:

For a mass located at $x_i$, set $a_{ii}$ to $-\omega^2 M$.

For a grounded spring at $x_i$, set $a_{ii}$ to $S$.

For a mass and spring at $x_i$, set $a_{ii}$ to $-\omega^2 M/(1-\omega^2 M/S)$.

A displacement constraint at $x_i$ is represented by setting the $i^{th}$ row and $i^{th}$ column of $[D]^{-1}$ to zero, and $a_{ii}$ to unity.

(d) Equivalent Forces

The system dynamic stiffness equation is inverted to give the plate displacements, $\{W\}$, which are inserted into the plate dynamic stiffness equation, Eq. 15, to yield the equivalent forces, $\{F\}$. The acoustic pressure and fluid particle velocities can then be found, as described in Section 2.

## 5. NUMERICAL RESULTS

(a) General

Two Fortran programs have been written which together compute and plot the intensity vectors. The first program calculates and stores on a disk file the pressure and particle velocities, at points on a selected grid in the positive $(x,z)$ quadrant, due to a single line force excitation. The second program uses this information to compute and plot the acoustic intensity vectors due to the plate with line attachments. Examples of the graphical output are shown in Figures 2-7 for which the following values in SI units were used:

plate: $E = 19.5E10$, $\nu = 0.29$, $\rho_s = 7700.0$, $h = 0.02$

fluid: $\rho = 1000.0$, $c = 1500.0$

Plate damping is represented by a complex Young's modulus, $E(1-i\eta)$, where $\eta$, the loss factor, is chosen as 0.02. The plotted vector lengths are proportional to $\bar{I}(x,z)$ in Figure 5, and to $\sqrt{[\bar{I}(x,z)]}$ in the other intensity plots.

(b) Single Line-Force Excitation, 250 Hz

Figures 2 and 3 show the intensity plots which illustrate the essential acoustic features due to low frequency excitation. First, the sound radiation is very inefficient. Secondly, the intense near field shows areas where energy is being received by the plate. Thirdly, the fluid surface wave associated with the subsonic plate flexural wave is evident. Finally, a closed loop of energy circulation can be seen.

(c) Single Line-Force Excitation, 2.5 kHz

Figure 4 shows the intensity plot at a mid-frequency which illustrates the increasing radiation efficiency of the plate.

(d) Single Line-Force Excitation, 25 kHz

Figure 5 shows the intensity plot at a high frequency. At this frequency, which is approximately twice the so-called critical frequency, the 'leaky' wave beam [6] can be seen emanating from the plate surface. The attenuation along the beam is small.

(e) Effect of Constraints, 250 Hz

Figures 6 and 7 show the vectors due to a force located between symmetrically placed displacement constraints. In Figure 6 the system is at its fundamental resonance: the top row of intensity vectors shows radiation coming from the region of the constraint. In Figure 7 the constraints cause intense near-fields with apparently little radiation: it is interesting to note that energy enters the plate at the drive point.

## 6. ACKNOWLEDGEMENTS

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WJS/jms

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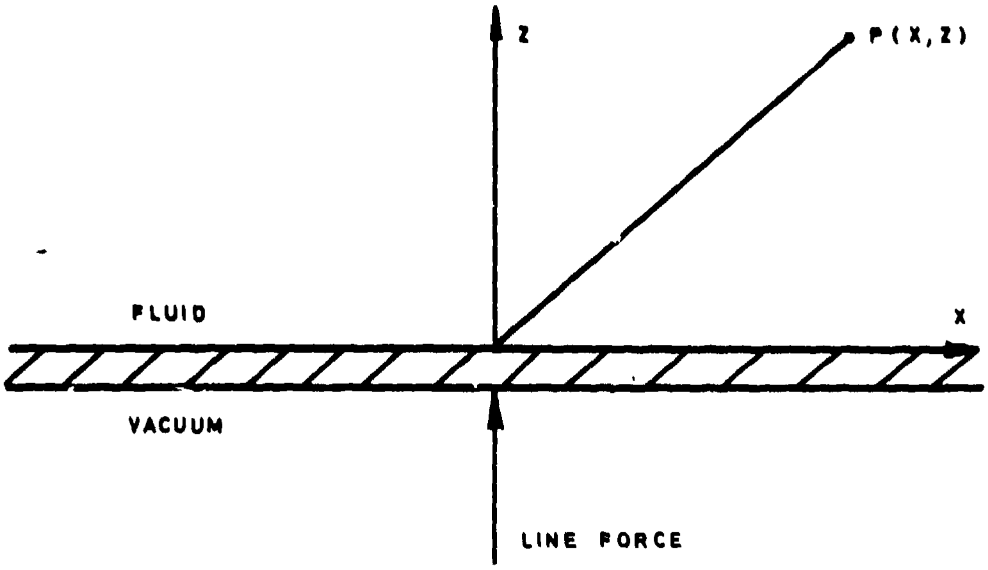


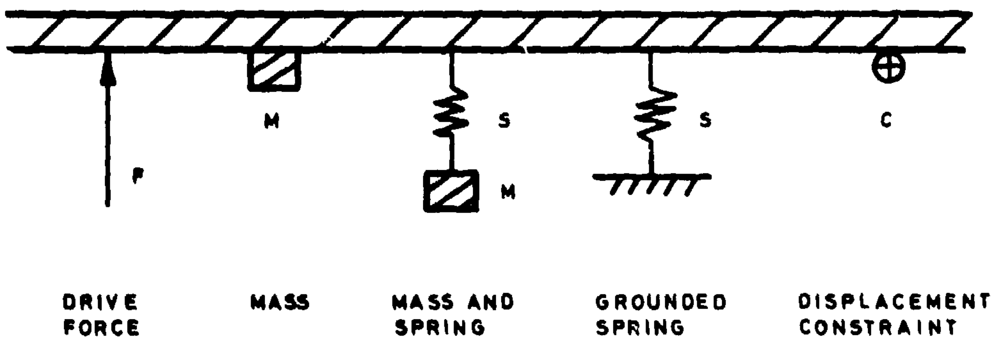




FIG. 1 PLATE COORDINATE SYSTEM AND ATTACHMENTS

FIG. 2 250Hz DRIVE FORCE AT X=0.0

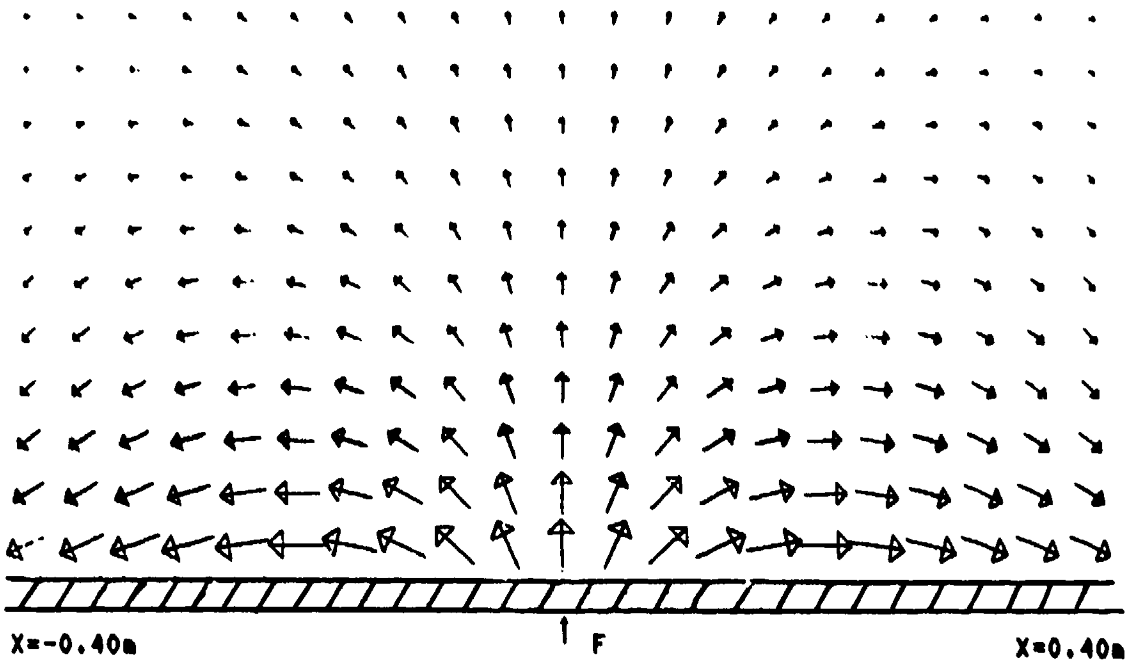



FIG. 3 250Hz DRIVE FORCE AT X=0.0

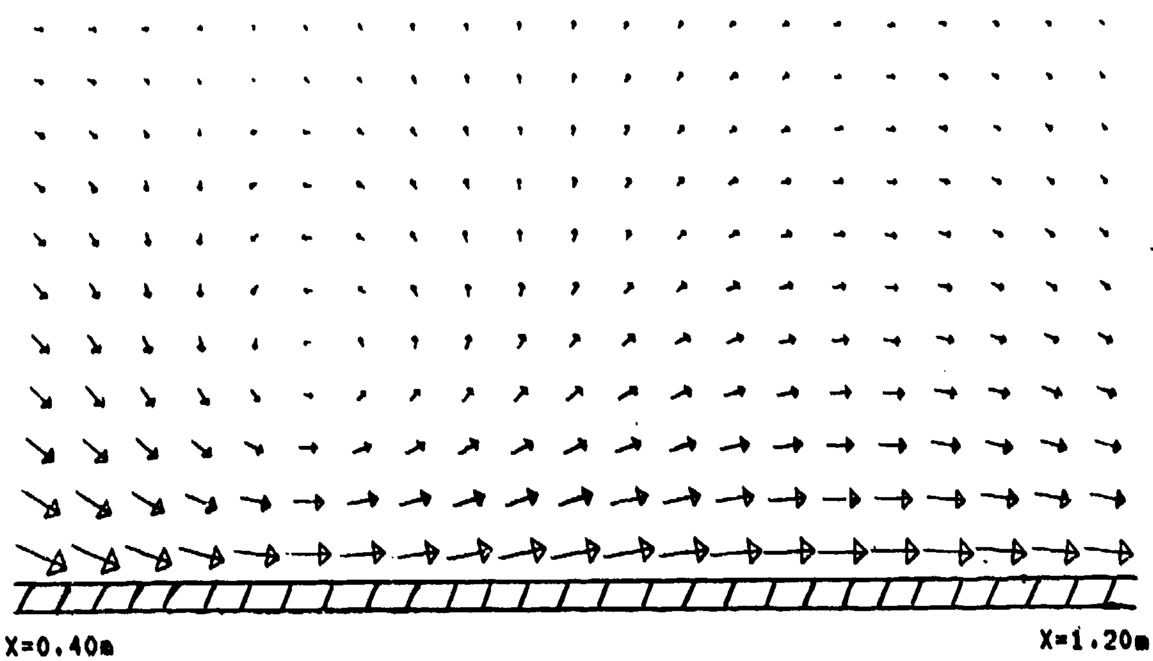


FIG. 4 2.5kHz DRIVE FORCE AT X=0.0

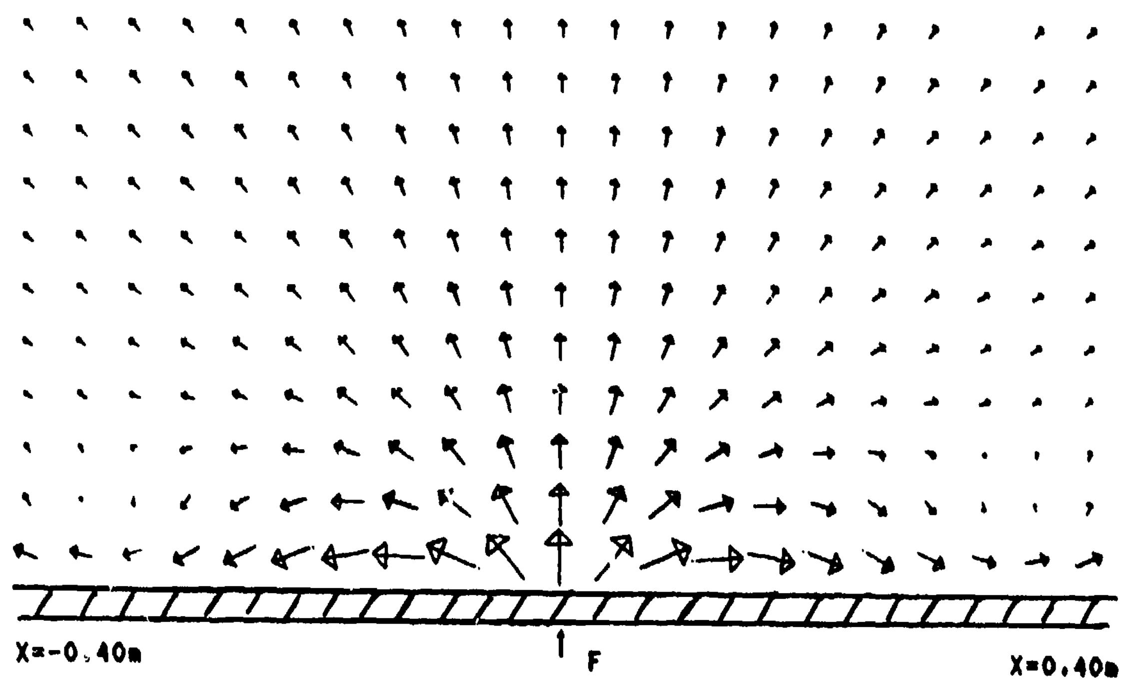



FIG. 5 25kHz DRIVE FORCE AT X=0.0

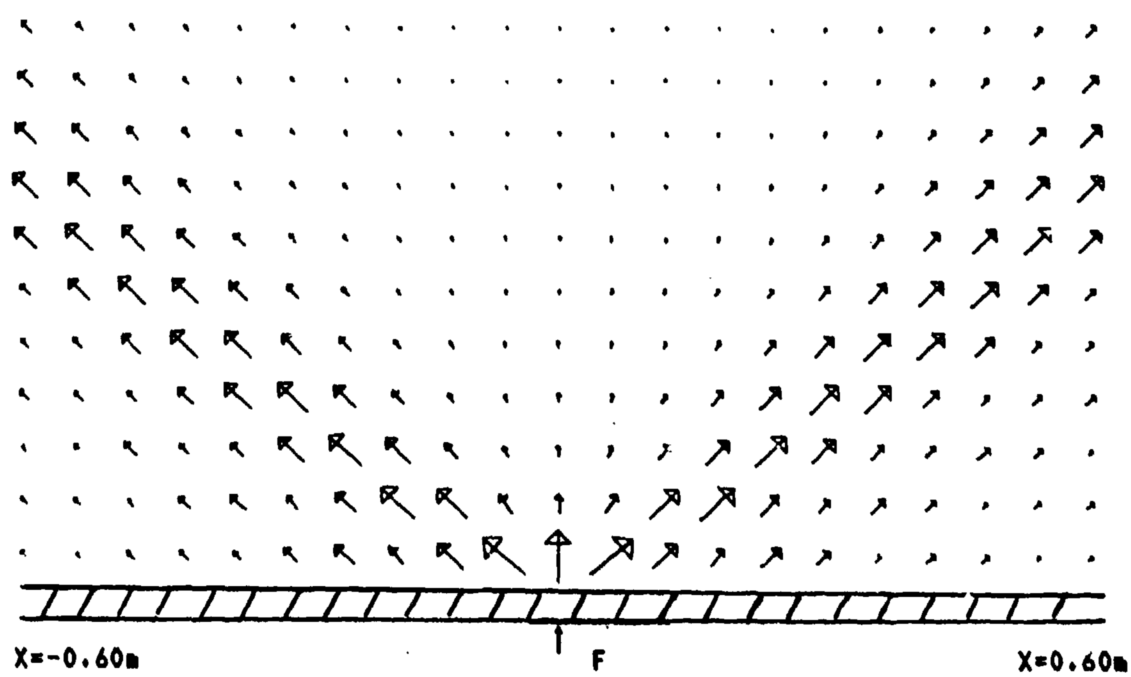


FIG. 6 DISPLACEMENT CONSTRAINTS AT ±0.2m FROM DRIVE FORCE, 250Hz

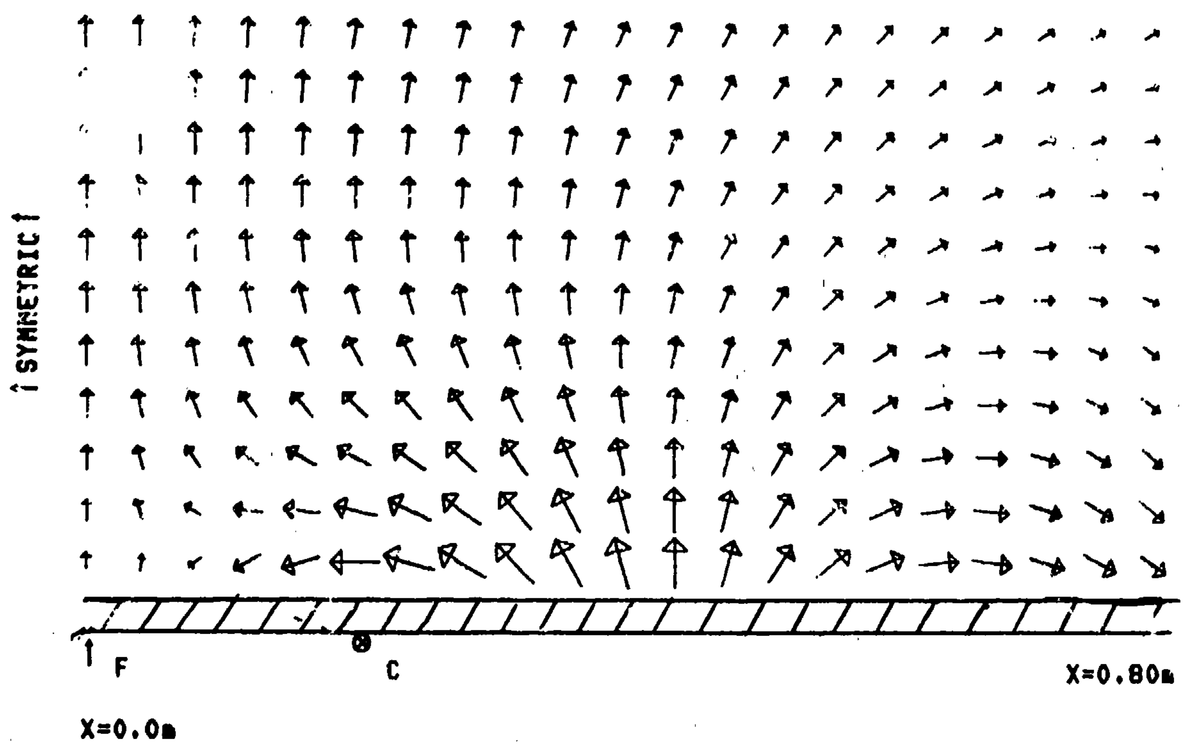
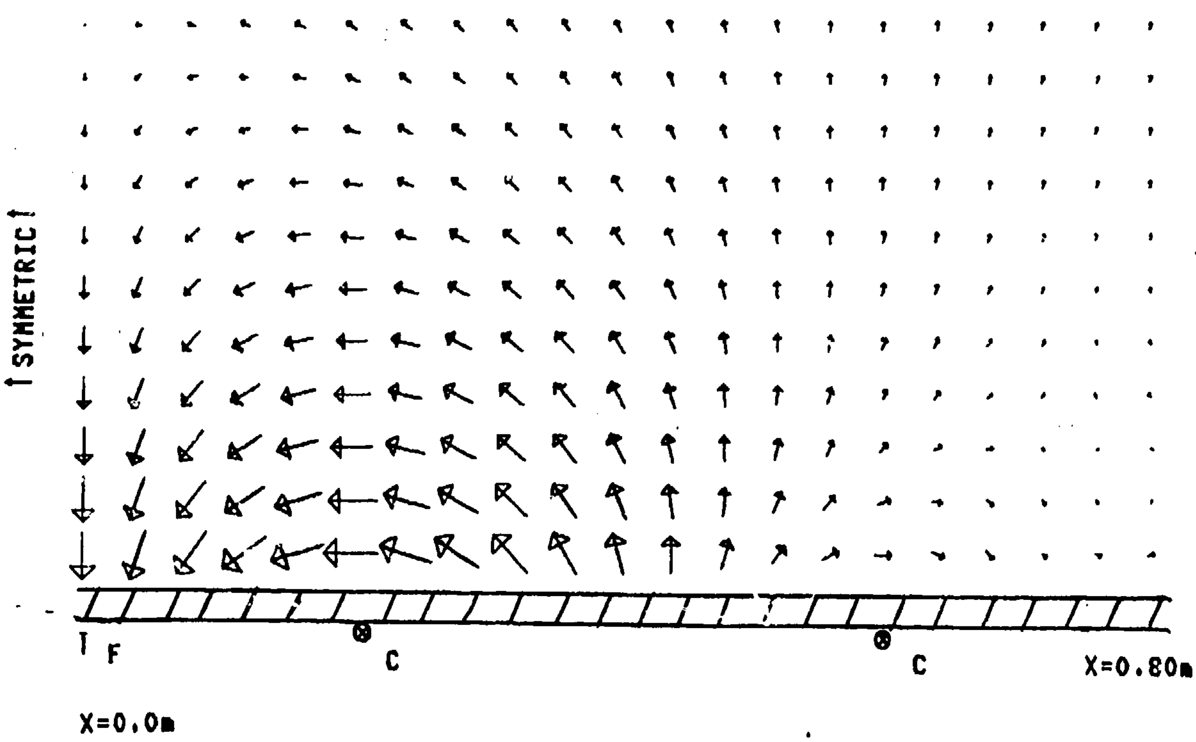



FIG. 7 DISPLACEMENT CONSTRAINTS AT ±0.2m AND ±0.6m FROM DRIVE FORCE, 250Hz



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